July 17, 2000 BI-TP 2000/25

Deconfinement through Chiral Symmetry Restoration in Two-Flavour QCD

S. Digal, E. Laermann and H. Satz

Fakultät für Physik, Universität Bielefeld
D-33501 Bielefeld, Germany

Abstract:

We propose that in QCD with dynamical quarks, colour deconfinement occurs when an external field induced by the chiral condensate strongly aligns the Polyakov loop. This effect sets in at the chiral symmetry restoration temperature T_χ and thus makes deconfinement and chiral symmetry restoration coincide. The predicted singular behaviour of Polyakov loop susceptibilities at T_χ is shown to be supported by finite temperature lattice calculations.

Finite temperature QCD at vanishing overall baryon density leads to two well-defined phase transitions. In the limit of infinite bare quark mass, for $m_q \to \infty$, one obtains pure $SU(N)$ gauge theory; here deconfinement occurs when the global center $Z_N \in SU(N)$ symmetry of the Lagrangian is spontaneously broken. The expectation value $L(T)$ of the Polyakov loop constitutes the order parameter for this transition, analogous to the magnetization $m(T)$ in Z_N spin theories. There exists a critical temperature T_d, with $L(T) = 0 \ \forall \ T \leq T_d$ and $L(T) > 0 \ \forall \ T > T_d$. The introduction of dynamical quarks ($m_q < \infty$) breaks this symmetry explicitly, and now $L(T) > 0 \ \forall \ T > 0$.

For $m_q = 0$, the chiral symmetry of the Lagrangian is spontaneously broken at low temperatures and restored for $T \geq T_\chi$. The chiral condensate $K(T) \equiv \langle \bar{\psi}\psi \rangle$ is the order parameter for this transition, with $K(T) > 0 \ \forall \ T < T_\chi$ and $K(T) = 0 \ \forall \ T \geq T_\chi$. A non-vanishing quark mass breaks chiral symmetry explicitly, and for large m_q, the temperature variation of the chiral condensate becomes completely smooth.

One might consider the inverse bare quark mass to play such a symmetry breaking role for deconfinement, which would imply that for $m_q \to 0$, the temperature variation of the Polyakov loop expectation value would also become smooth [1]. In lattice studies it is found, however, that this is not the case [2]: even for $m_q \to 0$, the Polyakov loop varies sharply with temperature and the corresponding susceptibility ($\langle L^2 \rangle - \langle L \rangle^2$) peaks sharply at the chiral restoration temperature T_χ, which is considerably lower than T_d. In somewhat loose terminology, one describes this situation by saying that in QCD for

$m_q \to 0$, deconfinement and chiral symmetry restoration coincide. The aim of this paper is to elucidate the underlying reasons for such behaviour and to show that chiral symmetry restoration in fact drives the observed Polyakov loop variation; in particular, it leads to singular behaviour for specific Polyakov susceptibilities.

Conceptually, our starting point is the idea that it is the inverse of the constituent quark mass m_Q, rather than the bare mass m_q, which acts as an external field for the Z_N symmetry [3]. Evidently there does not exist a clear definition of m_Q; for the present discussion, we take it to be determined by the (non-Goldstone) hadron masses. For our actual conclusions, however, we shall avoid the intermediate step of constituent quarks altogether. In the limit $m_q \to 0$, m_Q remains finite in the temperature region in which chiral symmetry is spontaneously broken, since the hadron masses do. If the hadron masses are related to the chiral condensate, e.g., if the nucleon mass is given by [4]

$$M_n \sim 3\ m_Q \sim \langle \bar{\psi}\psi \rangle^{1/3}, \tag{1}$$

then chiral symmetry restoration with $\langle \bar{\psi}\psi \rangle \to 0$ will lead to a sudden increase of the external field $H \sim 1/m_Q$, forcing a large explicit breaking of Z_N and hence a corresponding alignment of the Polyakov loop. We want to argue that it is this effect which causes a sharp variation of the Polyakov loop at $T_\chi < T_d$, with the functional form of the variation determined by chiral symmetry restoration, which is in general different from that obtained in pure gauge theory at T_d.

We begin by summarizing the main features of the critical behaviour for the two transitions, based on QCD with two species of dynamical quarks and colour $SU(3)$. The order parameter for chiral symmetry restoration, $K \equiv \langle \bar{\psi}\psi \rangle$, vanishes for $m_q = 0$ as $T \to T_\chi$ from below,

$$K(T, m_q = 0) \sim (T_\chi - T)^\beta, \quad T \lesssim T_\chi \tag{2}$$

while at $T = T_\chi$, it vanishes as

$$K(T_\chi = 0, m_q) \sim m_q^{1/\delta} \tag{3}$$

when $m_q \to 0$. The chiral quark mass susceptibility

$$\chi_m^K(T, m_q) = \left(\frac{\partial K}{\partial m_q}\right)_T \sim\ \langle (\psi\bar{\psi})^2 \rangle - \langle \psi\bar{\psi} \rangle^2\ \sim\ |T - T_\chi|^{-\gamma} \tag{4}$$

diverges at $T = T_\chi$ for $m_q \to 0$. The critical behaviour at the chiral symmetry restoration point is thus specified in terms of critical exponents $\beta,\ \delta,\ \gamma,\ ....$ For two massless quark flavours, the transition is conjectured to belong to the $O(4)$ universality class [6], which would determine the exponents. Present lattice calculations cannot yet determine if this conjecture is correct [7, 8].

In the limit $m_q \to \infty$, we recover pure $SU(3)$ gauge theory, for which the deconfinement transition is of first order, as it is for the corresponding Z_3 spin theory [5]. The resulting discontinuity at $T = T_d$ persists in $L(T, m_q)$ for sufficiently large m_q; but below some quark mass value $0 < m_q^d < \infty$, $L(T, m_q)$ and all its derivatives presumably vary smoothly with temperature. We want to argue, however, that for $m_q \to 0$, certain derivatives of $L(T, m_q)$ will become singular.

As indicated above, we now take the Z_N structure of QCD to be that of Z_N spin theory in an external field H, determined by the constituent quark mass. From Eq. (1), H and hence also the Polyakov loop thus become functions of K: the quark mass dependence of L enters through $K(T, m_q)$, with $L(T, K)$. In spin systems with external field, the magnetization $m(T, H)$ is for $H \neq 0$ an analytic function of T and H. We therefore assume that $L(T, K)$ is for $\infty > K > 0$ an analytic function of T and K, with the quark mass dependence of L entering through $K(T, m_q)$.

As a consequence, the critical behaviour of the chiral condensate will be reflected in the behaviour of the Polyakov loop at $T = T_\chi$. A first hint that this is the case is seen in [9]-[11]. For four flavours of light quarks, the chiral transition becomes first order, with a discontinuity in $K(T, m)$ at T_χ for $m_q < m_q^\chi$ smaller than some 'endpoint value' m_q^χ. The corresponding Polyakov loop $L(T)$ also shows a discontinuity at this temperature, as expected from our considerations. Note that here we have a first order transition at $T = T_\chi$ for $0 \leq m_q < m_q^\chi$, induced by chiral symmetry restoration, and another first order transition at $T = T_d > T_\chi$ for $m_q > m_q^d$, induced by spontaneous Z_N symmetry breaking. For $m_q^\chi < m_q < m_q^d$, $L(T, K)$ varies smoothly with T.

In the case of two dynamical quark flavours, the chiral transition is presumably continuous. From the L-variation

$$dL = \left(\frac{\partial L}{\partial T}\right)_K dT + \left(\frac{\partial L}{\partial K}\right)_T dK \tag{5}$$

we then find that the Polyakov loop susceptibilities

$$\chi_m^L = \left(\frac{\partial L}{\partial m_q}\right)_T = \left(\frac{\partial L}{\partial K}\right)_T \left(\frac{\partial K}{\partial m_q}\right)_T \tag{6}$$

and

$$\chi_T^L = \left(\frac{\partial L}{\partial T}\right)_{m_q} = \left(\frac{\partial L}{\partial T}\right)_K + \left(\frac{\partial L}{\partial K}\right)_T \left(\frac{\partial K}{\partial T}\right)_{m_q}, \tag{7}$$

must diverge at $T = T_\chi$ in the chiral limit $m_q = 0$, since the chiral susceptibilities $\chi_m^K = (\partial K/\partial m_q)_T$ and $\chi_T^K = (\partial K/\partial T)_{m_q}$ diverge in this limit.

In [2], the chiral susceptibilities χ_m^K and χ_T^K were studied on an $8^3 \times 4$ lattice for quark masses $m_q a = 0.075,\ 0.0375$ and 0.02. In Figs. 1 and 2, the results are shown as functions of the effective temperature variable $\kappa = 6/g^2$, where g denotes the coupling in the QCD Lagrangian. The increase of the peak height for decreasing quark mass indicates the divergence in the chiral limit $m_q \to 0$. In Figs. 3 and 4, we show the corresponding results for the Polyakov loop susceptibilities. They also peak sharply, the peak positions coincide with those for the chiral susceptibilities, and here as well the peak height increases with decreasing quark mass. The observed behaviour therefore provides clear support for the divergence of the temperature and quark mass derivatives of the Polyakov loop at T_χ in the chiral limit.

A direct comparison of the functional behaviour, ideally of the relevant critical exponents, for the two cases becomes difficult for two reasons. The chiral susceptibilities do not at present lead to the predicted $O(4)$ exponents; this may indicate that the quark masses

are still too large for pure critical behaviour. The Polyakov loop susceptibilities contain in addition unknown non-singular factors $(\partial L/\partial T)_K$ and $(\partial L/\partial K)_T$, which will modify the non-singular m_q-dependence relative to that of the chiral susceptibilities. Lattice studies for smaller quark masses would therefore be very helpful.

The 'temperature' susceptibilities χ_κ^K and χ_κ^L have also been studied on larger lattices $12^3 \times 4$ and $16^3 \times 4$, for the same three quark mass values [8]. In Figs. 5 and 6 we show the dependence of the peak height on the spatial volume. The two susceptibilities show a very similar peak increase in going from 8^3 to 12^3, but then saturation, indicating that there are no further finite size effects for the given quark mass value.

In summary, we have shown that chiral symmetry restoration, with the resulting sudden change in an effective constituent quark mass, leads to a suddenly increasing external field which aligns the Polyakov loops at $T = T_\chi < T_d$ and thus produces a strong explicit breaking of the Z_N symmetry of the gauge field part of the Lagrangian. This effect makes chiral symmetry restoration 'coincide' with deconfinement. The resulting predictions for diverging Polyakov loop susceptibilities are found to be well supported by finite temperature lattice calculations for full QCD with two flavours of light quarks.

Acknowledgements

It is a pleasure to thank F. Karsch, P. Petreczky and A. M. Srivastava for helpful discussions, and we are grateful for the financial support by the German Science Ministry BMBF under contract 06BI902.

References

[1] P. Hasenfratz, F. Karsch and I. Stamatescu, Phys. Lett. 133 B (1983) 221.

[2] F. Karsch and E. Laermann, Phys. Rev. D 50 (1994) 6954.

[3] R. V. Gavai, A. Gocksch, and M. Ogilvie, Phys. Rev. Lett. 56 (1986) 815.

[4] B. L. Ioffe, Nucl. Phys. B 188 (1981) 317.

[5] B. Svetitsky and L. G. Yaffe, Nucl. Phys. B 210 [FS6] (1982) 423.

[6] R. Pisarski and F. Wilczek, Phys. Rev. D29 (1984) 339.

[7] S. Aoki et al. (JLQCD), Phys. Rev. D 57 (1998) 3910.

[8] For a survey, see E. Laermann, Nucl. Phys. B (Proc. Suppl.) 63 A-C (1998) 114.

[9] R. Gupta et al., Phys. Rev. Lett. 57 (1986) 2621.

[10] F. Karsch et al., Phys. Lett. 188 B (1987) 353.

[11] R. V. Gavai et al. (MTc), Phys. Lett. B 241 (1990) 567.

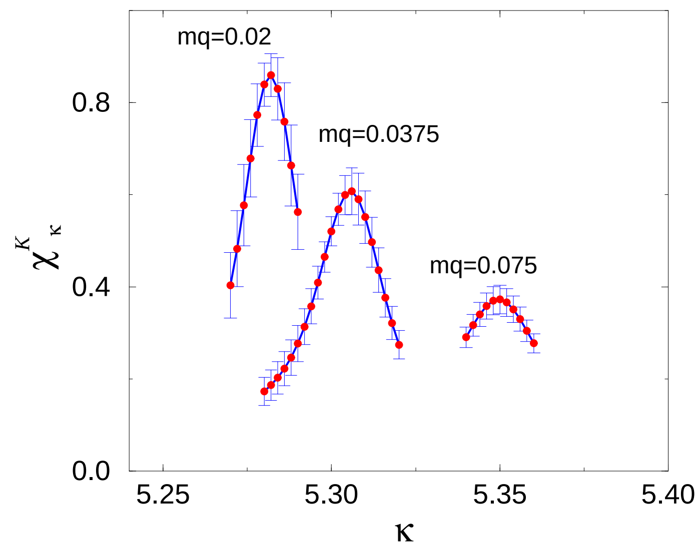

Figure 1: The chiral temperature susceptibility χ^K_κ as function of the temperature variable $\kappa = 6/g^2$.

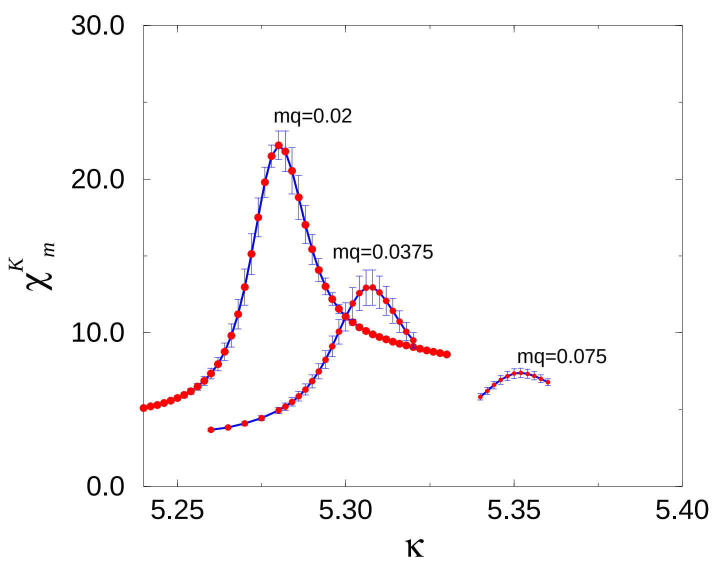

Figure 2: The chiral quark mass susceptibility χ^K_m as function of the temperature variable $\kappa = 6/g^2$.

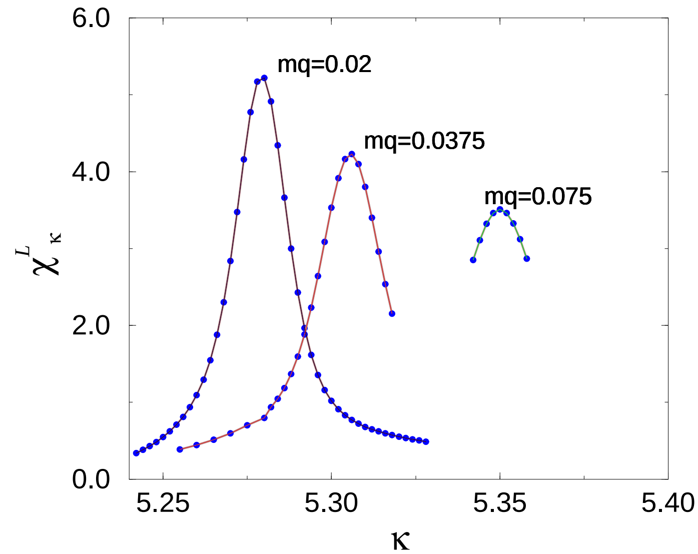

Figure 3: The Polyakov loop temperature susceptibility χ_κ^K as function of the temperature variable $\kappa = 6/g^2$.

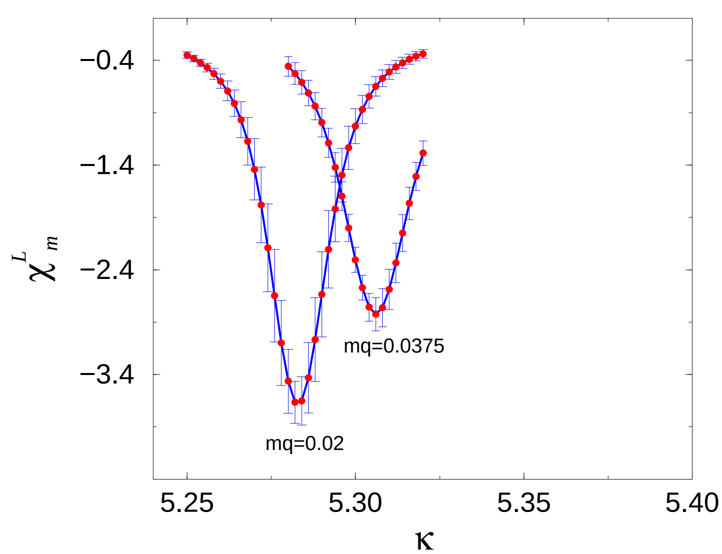

Figure 4: The Polyakov loop quark mass susceptibility χ_m^K as function of the temperature variable $\kappa = 6/g^2$.

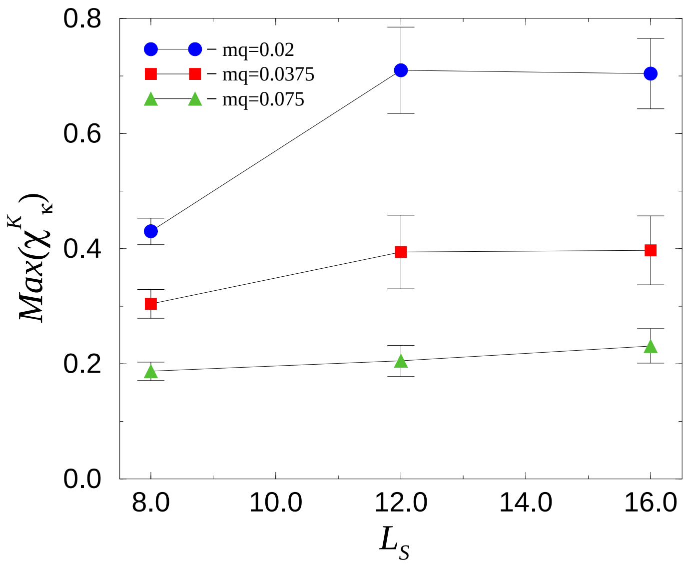

Figure 5: The dependence of the χ_{κ}^{K} peak height on spatial lattice size.

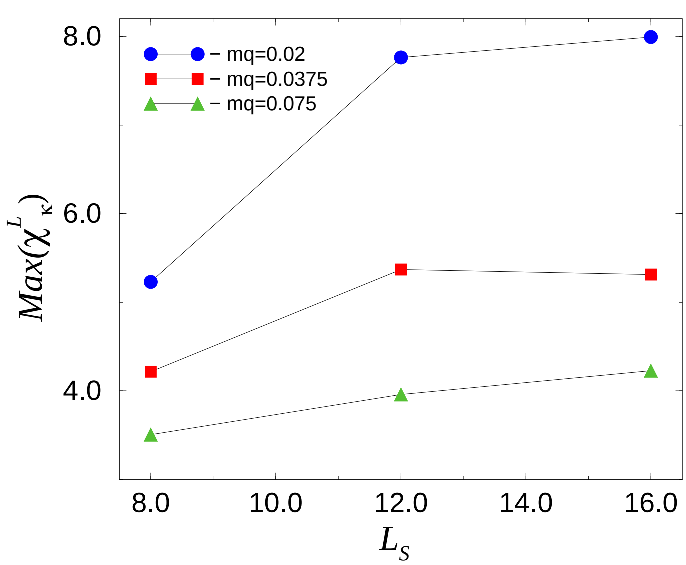

Figure 6: The dependence of the χ_{κ}^{L} peak height on spatial lattice size.